Math 115

Spring 2017

Lecture 23

<u>Ch. 6</u> : Rational Expressions

1) what is a Rational Expression?

$$\frac{\text{Polynomial}}{\text{Polynomial}}$$

$$\frac{2x+1}{x^2-4}, \quad \frac{x^2+3x-4}{x^2-x-2}, \quad \frac{3x^2+5x-7}{2x^2-3x+4}$$

2) How to Simplify a rational expression:

a) factor numerator & denominator completely

b) Cross-out any common factor

Simplify $\dfrac{x^2-4}{x^2+3x+2} = \dfrac{(x+2)(x-2)}{(x+1)(x+2)} = \boxed{\dfrac{x-2}{x+1}}$

Simplify: $\dfrac{x^2 - 3x - 28}{x^2 - 16} = \dfrac{(x-7)\cancel{(x+4)}}{\cancel{(x+4)}(x-4)}$

$$= \boxed{\dfrac{x-7}{x-4}}$$

Simplify: $\dfrac{3x^2 - 2x - 5}{x^2 - 4x - 5} = \dfrac{(3x-5)\cancel{(x+1)}}{(x-5)\cancel{(x+1)}}$

$$= \boxed{\dfrac{3x-5}{x-5}}$$

Restricted Values or Excluded Values are those values for which the denominator becomes zero.

How to find excluded values:

a) Denominator = 0

b) Solve

Find all excluded values:

$\dfrac{2x - 5}{x+4} \longrightarrow x + 4 = 0$ $\boxed{x = -4}$ -4 is an excluded value.

find all <u>excluded values</u>

$$\frac{x-7}{x^2-2x-24}$$

$x^2-2x-24=0$

$(x-6)(x+4)=0$

by Z.F.R.

$x-6=0$ or $x+4=0$

$\boxed{x=6}$ $\boxed{x=-4}$

So E.V. are 6 & -4.

find all <u>excluded values</u>

$$\frac{3x-2}{x^2-25}$$

E.V.: ±5

$x^2-25=0$

$(x+5)(x-5)=0$

$x=-5$ $x=5$

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① Simplify: $\dfrac{x^2+6x+9}{x^3+27} = \dfrac{\cancel{(x+3)}(x+3)}{\cancel{(x+3)}(x^2-3x+9)}$

A^3+B^3

$$= \boxed{\dfrac{x+3}{x^2-3x+9}}$$

② find all <u>excluded values</u> for $\dfrac{x^2-3x+5}{3x^2-4x-15}$

$3x^2-4x-15=0$

$3x^2-9x+5x-15=0$ $\overbrace{}^{}$ P=-45

-45 S=-4

$3x(x-3)+5(x-3)=0$ -9 & 5

$(x-3)(3x+5)=0$

$x-3=0$ $3x+5=0$

$\hookrightarrow x=3$ $\hookrightarrow x=-5/3$

E.V. are 3 & -5/3

How to multiply Rational Expressions:

a) factor numerators & denominators completely.

b) Cross-out any common factors vertically or diagonally.

c) Remaining factors, write them as product. Do not multiply them out.

Multiply:

$$\frac{x^2-4}{x^2+5x+6} \cdot \frac{x^2-9}{x^2-2x} = \frac{(x+2)(x-2)}{(x+2)(x+3)} \cdot \frac{(x-3)(x+3)}{x(x-2)}$$

$$= \boxed{\frac{x-3}{x}}$$

Multiply:

$$\frac{x^2+7x+6}{x^2+6x+5} \cdot \frac{x^2-10x+25}{x^2+12x+36}$$

$$= \frac{(x+6)(x+1)}{(x+5)(x+1)} \cdot \frac{(x-5)(x-5)}{(x+6)(x+6)} = \boxed{\frac{(x-5)^2}{(x+5)(x+6)}}$$

Multiply:

$$\frac{x^3-64}{x^2-16} \cdot \frac{x^2+4x}{x^2+4x+16} = \frac{(x-4)(x^2+4x+16)}{(x+4)(x-4)} \cdot \frac{x(x+4)}{x^2+4x+16}$$

$$= \boxed{x}$$

How to divide rational expression:

a) change $\div$ to $\cdot$

b) Multiply by the reciprocal of the rational expression which was after $\div$.

c) Now factor & simplify.

Divide

$$\frac{x^2-16}{x^2-9} \div \boxed{\frac{x^2-7x+12}{x^2+7x+12}} = \frac{(x+4)(x-4)}{(x+3)(x-3)} \cdot \frac{(x+4)(x+3)}{(x-4)(x-3)}$$

$$= \boxed{\frac{(x+4)^2}{(x-3)^2}}$$

Divide: $\dfrac{2x^2-11x+5}{5x-25} \div \dfrac{4x-2}{10}$

$$= \frac{(2x-1)(x-5)}{5(x-5)} \cdot \frac{\overset{2}{10}}{2(2x-1)} = \frac{2}{2} = \boxed{1}$$

Divide:

$$\frac{8x^2-18}{2x^2-5x+3} \div \frac{6x^2+7x-3}{x^2-9x+8}$$

$$= \frac{2(2x+3)(2x-3)}{(2x-3)(x-1)} \cdot \frac{(x-1)(x-8)}{(2x+3)(3x-1)} = \boxed{\frac{2(x-8)}{3x-1}}$$

Solve:

① $2x^2 - x = 28$

$2x^2 - x - 28 = 0$

$(2x + 7)(x - 4) = 0$

$2x + 7 = 0 \qquad x - 4 = 0$

$x = \dfrac{-7}{2} \qquad\quad x = 4$

$\left\{ \dfrac{-7}{2}, 4 \right\}$
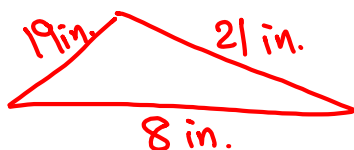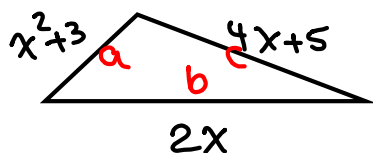
② $x^3 = 16x$

$x^3 - 16x = 0$

$x(x^2 - 16) = 0$

$x(x + 4)(x - 4) = 0$

$\boxed{x = 0} \qquad x + 4 = 0 \qquad x - 4 = 0$

$\boxed{x = -4} \qquad \boxed{x = 4}$

$\{0, \pm 4\}$

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find all 3 sides of the triangle below if the perimeter is 48 inches

$x^2 + 3$ a $4x + 5$ b c $2x$

19 in.   21 in.   8 in.

$P = 48$

$a + b + c = 48$

$x^2 + 3 + 2x + 4x + 5 = 48$

$x^2 + 6x - 40 = 0$

$(x + 10)(x - 4) = 0$

$\cancel{x = -10} \qquad x = 4$
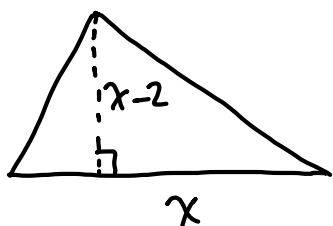
Hint: $A = \dfrac{bh}{2}$

$A = 60\ \text{in}^2$

Find base & height

base   12 in.

height   10 in.

$\dfrac{x(x-2)}{2} = 60$

$LCD = 2$

$x(x-2) = 120$

$x^2 - 2x - 120 = 0$

$(x+10)(x-12) = 0$

$x = -10$ $x = 12$

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find x:
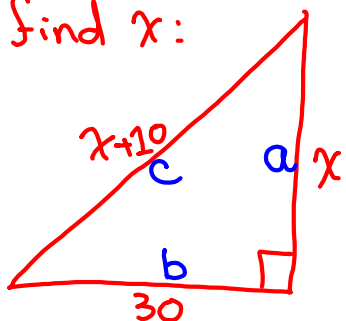
Right Triangle

Pythagorean thm

$a^2 + b^2 = c^2$

$x^2 + 30^2 = (x+10)^2$

$x^2 + 900 = (x+10)(x+10)$

$x^2 + 900 = x^2 + 10x + 10x + 100$

$900 - 100 = 20x$

$800 = 20x$

$\boxed{x = 40}$

In a right triangle, one leg is 12 ft more than another leg.

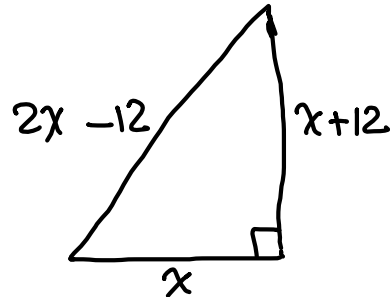The hypotenuse is 12 ft less than twice the shorter leg.

Find all three sides.

Use pythagorean thrm:

$$a^2 + b^2 = c^2$$

$$x^2 + (x+12)^2 = (2x-12)^2$$

$$\cancel{x^2} \cancel{+ x^2} + 24x + \cancel{144} = \cancel{4x^2} - 48x + \cancel{144}$$

$$0 = 2x^2 - 72x$$

$2x - 12$ $x + 12$

x

$$2x^2 - 72x = 0$$

Divide by 2 to reduce numbers only

$$x^2 - 36x = 0$$

$$x(x - 36) = 0$$

$$x = 0$$

$$x = 36$$

60 ft.

$2x-12$ 48ft $x+12$
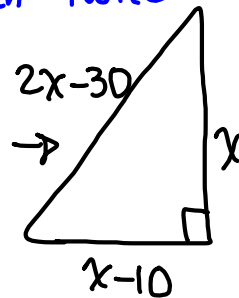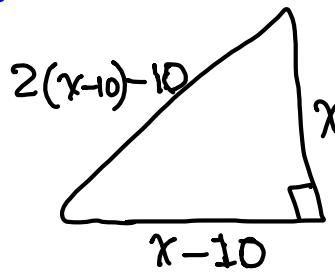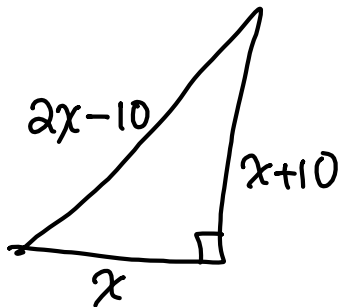
36 ft

x
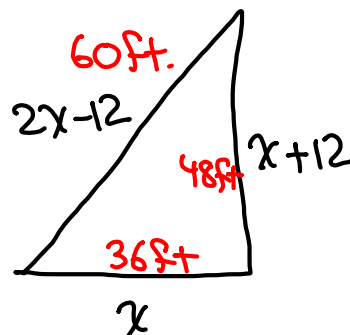
To verify

$$36^2 + 48^2 = 60^2$$

In a right triangle, one leg is 10 cm less than another leg.
The hypotenuse is 10 cm less than twice the shorter leg. Find all 3 sides.

$2(x-10)-10$ x → $2x-30$ x

$x-10$ $x-10$

Redraw

$2x-10$ $x+10$ x

$x^2 + (x+10)^2 = (2x-10)^2$

$x^2 + x^2 + 20x + \cancel{100} = 4x^2 - 40x + \cancel{100}$

$0 = 2x^2 - 60x$

$2x^2 - 60x = 0$

Divide by 2

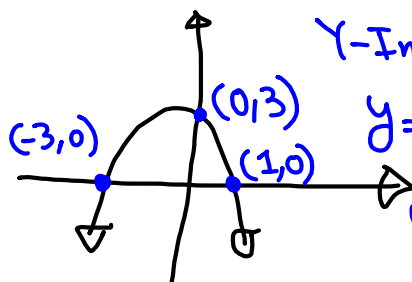$x^2 - 30x = 0$

$x(x-30) = 0$

$\cancel{x=0}$ → $x = 30$

$50cm$ $2x-10$ $40cm$ $x+10$

$30cm$ x
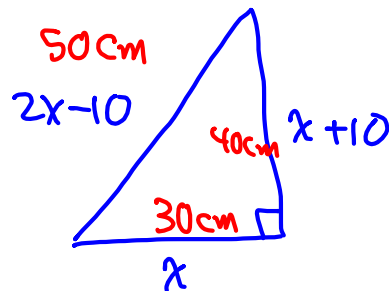
The graph for $y = -x^2 - 2x + 3$ is given below. Find all intercepts.

Hint

Y-Int: $(0, \quad)$

X-Int: $(\quad, 0)$

$(\quad, 0)$

Y-Int → $x = 0$

$y = -0^2 - 2(0) + 3$

$= 3$

$\boxed{\text{Y-Int } (0, 3)}$

x-Int. → $y = 0$

$\boxed{\begin{array}{l} \text{x-Ints:} \\ (-3, 0), \\ (1, 0) \end{array}}$

$x = -3$

$x = 1$

$-x^2 - 2x + 3 = 0$

Multiply by -1

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

Class Qz

Solve:

① $(x-7)(3x+5) = 0$

② $3x^2 + 4 = 13x$

③ $x^3 = 25x$